Housing Appreciation (Depreciation) and Owners’ Welfare:
An Alternative view∗

Fu-Chuan Lai† and David Merriman‡

September 23, 2009

Abstract. The Frank’s (2006) model that a house owner is better off whenever the housing price changes is mathematically generalized to include property taxes, mortgage loan, and moving costs in this paper. It is shown that the Frank’s conclusion is valid only when there is no property tax, no mortgage loan, and no moving costs. If the property tax is introduced and when the house owner adjusts his/her house size in response to a price change, the owner will be better off only when there is a big appreciation or depreciation, while he/she may not be better off even under a big depreciation when the mortgage loan is further considered.

∗We would like to thank John McDonald, Nathan Anderson, Richard Dye, Kristin Munro, and Richard M. Peck for their valuable comments. All remaining errors are ours.

†Department of Economics, National Taipei University, 151, University Rd., San-Shia, Taipei, 237, Taiwan. Tel.: +886-2-8674-7165, Fax: +886-2-2673-9880, E-mail: uiuclai@mail.ntpu.edu.tw

‡Institute of Government and Public Affairs, University of Illinois, 815 W. Van Buren Street, Suite 525, Chicago, IL 60607MC-191, Phone: (312) 996-1381, Fax: (312) 996-1404, and Department of Public Administration, University of Illinois at Chicago, Room 138, 412 S. Peoria Street, Chicago, IL 60607MC-278, Phone: (312) 355-2672, Fax: (312) 996-8804, E-mail: dmerrim@uic.edu.
Housing Appreciation (Depreciation) and Owners’ Welfare: An Alternative view

Fu-Chuan Lai and David Merriman

1 Introduction

In his Microeconomics textbook, Frank (2006) simply shows that starting from a fixed bundle of housing and a composite good, either an increase in the price of housing (appreciation) or a decrease in the housing price (depreciation) makes an owner better off (pp. 166-7). It is quite intuitively for the appreciation case, but not for the depreciation case. One parallel research on this issue, Lai, McDonald, and Merriman (2009) (LMM model, hereafter),1 use a diagrammatic analysis and a lifetime span consideration, shows that a large moving cost or mortgage balance may cause the owners worse off in an appreciation case, while it can never be worse off in a depreciation case if the owner intends to stay in his/her house.2

However, the property tax is an annual tax and is proportional to the housing value and thus the tax payment can be changed year after year due to a housing price change. Therefore the composite good consumption is also affected by the property tax change. Based on this

1 Other related literatures are Sinai and Souleles (2005) and Cauley et al. (2007).
2 The major difference between their paper and the current paper is in the definition of wealth when housing price changes and owners change their houses. Precisely speaking, in their model the new wealth after housing price changes form $p$ to $p'$ is $w' = p'(1+t)h_0 + c_0$, where $h_0$ is the house size before appreciation (depreciation) and $c_0$ is the composite good consumption before appreciation (depreciation), while the current paper is $w' = p'h_0 + c_0$. The reason of the current setting is because an owner expended $p(1+t)h_0$ to buy his/her house, while he/she can only receive $p'h_0$ when the house is sold. In other words, property tax is paid by the buyer, but never be received by the seller. Due to this setting, Figure 2 and Figure 4 in their paper show that when $p'/p = 1$, the utility ratio $v'/v_0 = 1$, while in our results, $v'/v < 1$ whenever $p'/p = 1$. Another difference between this two papers is that they treat the remaining mortgage loan as a constant cost (something like a moving cost), while the current paper assumes that the mortgage loan is a percentage of the house value. Therefore, in their paper, if the depreciation is large enough, the owner is still better off; while in the current paper, a huge depreciation make the owner worse off if he/she pay off the loan and then buy a new house, because the loan is evaluated at a price $p$, while when he/she tries to sell a house, the house is evaluated at a price $p'$ which is much less than $p$. His/her utility may thus fall when as the housing size is adjusted.
observation, a mathematical modeling is developed in this paper and some numerical results are diagrammatic analyzed. A crucial difference between this paper and the LMM model is that they think that the wealth is tax included basing on a lifetime framework, while this paper defines the wealth coming from a house is the total money that the owner can receive when the house is sold, which is obviously not tax included. The tax is paid to the government and thus can not be counted as the owner’s wealth. Consequently, the property tax is something like a transaction cost whenever people buy or sell a house and it can thus limit the possibility of housing transitions. Therefore, only when there is a big appreciation or depreciation can raise the owner’s welfare.

Moreover, people usually use a mortgage loan to buy their houses in the real world, so the net wealth coming from a house for the owner should subtract the loan from the present value of the house. Obviously, when the house is appreciated, the net wealth coming from the house is raised, while if the house is depreciated, the owner encounters a capital loss. In a depreciation case, if a owner want to change his house, he must pay off the mortgage loan. But he may not want to do that, because his house value is no longer so valuable. If he/she did change to another house with the optimal size, he/she must also reduce a lot of composite good consumptions. Therefore, he must choose to stay in his current house. This view can explain why people feel worse off (instead of better off from Frank’s view) when a heavy slump happened in the housing market during the financial crisis in 2008.

The rest of the paper is organized as follows. Section 2 recapitulates Frank’s (2006) model, Section 3 revises the model to add property taxes, and then moving costs are considered in Section 4. Some concluding remarks are discussed on Section 5.

2 The Model

A Cobb-Douglas utility function will be employed hereafter as a standard example to depict our model mathematically. Suppose each agent has a utility function \( u = h^\alpha \cdot c^{(1-\alpha)} \), where \( h \) is the housing unit, \( c \) is the volume of a composite good (price normalized as one),\(^3\) and \( \alpha \) can be seen as the housing expenditure proportion. The agent’s budget constraint is \( w_0 = p(1+t-\beta) \cdot h + c \), where \( w_0 \) represents the initial wealth and \( p \) is the per unit house price, \( t \) is the property tax rate, and \( \beta \) is the mortgage percentage of the housing value. Using a monotonic transformation of the utility function such that \( \hat{u} = \ln u = \alpha \ln(h) + (1-\alpha) \ln(c) \) can simplify the solving process.

\(^3\)Readers can think \( c \) as another asset (money) which its price is always fixed at one.
Solving for the Lagrangian \( L = \hat{u} + \lambda(w_0 - p(1 + t - \beta)h - c) \) can yield
\[ h_0 = \frac{\alpha w_0}{p(1 + t - \beta)}, \quad c_0 = (1 - \alpha)w_0. \] (1)
The equilibrium indirect utility is then
\[ v_0 = h_0^\alpha \cdot c_0^{(1-\alpha)} = \alpha^\alpha \cdot (1 - \alpha)^{(1-\alpha)} \cdot w_0 \cdot \left( \frac{1}{p(1 + t - \beta)} \right)^\alpha. \] (2)
Suppose the housing price now changes to \( p' \), then the new wealth is \( w' = p' \cdot h_0 - \beta ph_0 + c_0 \).\(^4\) This is because his/her current housing value is now \( p' \cdot h_0 \), while his/her mortgage is still \( \beta ph_0 \). If the agent decides to change the house size, he/she must first pay off the old mortgage loan and apply for a new mortgage loan.\(^5\) The Lagrangian is then
\[ L = \hat{u} + \lambda(w' - p'(1 + t - \beta)h - c). \] (3)
Solving for the Lagrangian yields
\[ h' = \frac{\alpha w'}{p'(1 + t - \beta)}, \quad c' = (1 - \alpha)w'. \] (4)
Then the indirect utility function is
\[ v' = h'^\alpha \cdot c'^{(1-\alpha)} = \alpha^\alpha \cdot (1 - \alpha)^{(1-\alpha)} \cdot w' \cdot \left( \frac{1}{p'(1 + t - \beta)} \right)^\alpha. \] (5)
Note that the agent has another choice which is staying in his/her current house. The wealth in this case is \( w_A = w_0 - (p' - p)th_0 \), because the property tax will change as the housing price changes. The Lagrangian is\(^6\)
\[ L = \hat{u} + \lambda(w_A - p(1 + t - \beta)h_0 - c). \] (6)
\(^4\)Note that
\[ w' - w_0 = \frac{w_0 \alpha (p' - p(1+t))}{p(1 + t - \beta)} \geq 0 \quad \text{if} \quad p' \geq p(1 + t). \]
In other words, unless the new housing price exceeds the real purchase price (tax included), the owner does not have a wealth increase when housing price increases.
\(^5\)It is assumed that house size is fixed. For people want to adjust their house sizes to maximize their utility, the only feasible way is to sell the current house and buy a new house with an ideal size.
\(^6\)Note that \( w_0 \) is a stock concept, while property tax and composite good consumption are flow concepts. To accommodate the stock and flow variables in a static model, one can think that each period an agent gets an endowment \( w_0 \) and re-purchases the same house with the original conditions in \( p, t, \) and \( \beta \), even though the endowment and the purchase decision only happened once. In other words, readers can think this house is permanently rented. Moreover, the tax payment will vary due to the housing price changes. The increase of tax payment will accompany with a decrease on composite good consumption due to the owner staying in the same house.
Solving for the Lagrangian yields
\[ c_A = (1 - \alpha)w_A. \] (7)

Then the indirect utility function is then
\[ v_A = h^\alpha_0 \cdot c_A^{(1-\alpha)} = \left( \frac{w_0 \alpha}{p (1 + t - \beta)} \right)^\alpha \left( (1 - \alpha) \left( w_0 - \frac{(p - p) t \alpha w_0}{p (1 + t - \beta)} \right) \right)^{1-\alpha}. \] (8)

Although comparing \( v_0 \), \( v' \), and \( v_A \) directly is feasible, the forms are too messy. Therefore, a diagram analysis will be held hereafter.

First, suppose that \( w_0 = 100, t = 0, \alpha = 0.3, \beta = 0, p = 10 \), this can be seen as the Frank’s model, because there is no property tax, no mortgage loan, and no moving costs. As shown in Figure 1, \( v_0 \) and \( v_A \) are identical no matter how large the housing price changes, and \( v' \) is an \( U \) shape curve. Note that and \( v' \) is greater than \( v_0 \) whenever for \( p' \neq p = 10 \). Therefore, the Frank’s result that owners are better off either in appreciation or depreciation is thus verified.

![Figure 1: \( v_0 \), \( v' \), and \( v_A \) under Frank's model \( t = 0, \beta = 0 \)](image)

Second, if now a property tax is included \( (t = 0.02) \), and \( w_0 = 100, \alpha = 0.3, \beta = 0, p = 10 \) are all the same as before, then \( v_0, v', \) and \( v_A \) can be drawn as in Figure 2. Note that the \( v' \) curve intersects the \( v_0 \) curve twice. It means that a small appreciation or depreciation can not

---

*In general, people spend 30 percent of income on their housing service.*
make the owner better off. This is because the property tax is something like a transaction cost. If the gain from appreciation or depreciation cannot cover the transaction cost of buying or selling a house, the owner is then worse off. Moreover, note that Frank’s model is assuming that people change houses whenever \( p' \neq p \). He did not consider the indirect utility for those people who choose to stay in the same house. It is shown by the green line that when \( p' < p \) (depreciation), staying in the same house yields a higher utility than moving to a new house. This is because when the housing price decreases, if he/she stays in the same house, the property tax is also decreased and he/she can increase his composite good consumptions. In other words, if housing depreciated, it means that the agent has more disposable income and yields a higher indirect utility when he/she stays in the same house. For the appreciation case, the situation is reversed.

![Figure 2: \( v_0, v', \) and \( v_A \) under \( t = 0.02, \beta = 0 \)](image)

Third, in the real world most people buy their housing by using a mortgage loan. If the loan equals 80% of the house value (\( \beta = 0.8 \)), and other things being the same (\( w_0 = 100, t = 0.02, \alpha = 0.3, p = 10 \)), then \( v_0, v' \), and \( v_A \) can be drawn as in Figure 3. It is shown that \( v_0 \) and \( v_A \) are similar to that of in Figure 2, while \( v' \) is very different from that of in Figure 2. When the house is depreciated, changing to a new house is totally worse off for those agents in Figure 3. This is because that the mortgage loan is still the same as before, but his/her current housing value is no longer as so high. He/she must pay off the loan before his/her buying a new and larger house. It is thus that staying in the same house is his/her best choice.
when housing price decreases. On the other hand, when housing price has a large increase, changing to a new (and smaller) house is his/her best choice.

Figure 3: $v_0$, $v'$, and $v_A$ under $t = 0.02$, $\beta = 0.8$

Fourth, if the government raises the property tax rate from 0.02 to 0.15, other things being the same ($w_0 = 100$, $\alpha = 0.3$, $\beta = 0.8$, $p = 10$), then $v_0$, $v'$, and $v_A$ can be drawn as in Figure 4. Note that the property tax is a transaction cost for the housing buyers, and thus when the tax rate increases, all $v_0$, $v_A$, and $v'$ falls (comparing with Figure 3). In other words, the higher property tax rate, the lower housing transactions.

Fifth, if the agent spends 60 percent ($\alpha = 0.6$) of wealth on housing, and $w_0 = 100$, $t = 0.02$, $\beta = 0.8$, $p = 10$ are all the same as before, then $v_0$, $v'$, and $v_A$ can be drawn as in Figure 5. Comparing with Figure 3, the overall indirect utility functions in Figure 5 have similar shapes, but with lower values. This is because the higher proportion of expenditure on housing, the more impact to the utility when housing depreciates.

Sixth, if the property tax rate varies from 0.01 to 0.25, and $w_0 = 100$, $\beta = 0.8$, $p' = 12$, and $\alpha = 0.3$, then $v_0$, $v'$, and $v_A$ can be drawn as in Figure 6. The higher property tax rate, the less gains for changing to a new house, and when the property tax rate is very high, changing to a new house can no longer be a best choice for a house owner. All the above results can be summarized as the follows.

**Simulation results.** (1) Frank (2006) is correct when there is no property tax and no mortgage
loans \((t = 0, \text{ and } \beta = 0)\) \((2)\) When the property tax is included \((t = 0.02 \text{ and } \beta = 0)\), Frank’s conclusion can only be applied in a big change of housing prices. When housing price falls (rises), house owners are better (worse) off due to the less (higher) property tax payment. \((3)\) When people buy their houses by using a mortgage loan \((\beta = 0.8)\), then Frank’s conclusion is
correct only for those housing prices have a big appreciation, while housing owners are worse off when the housing price has a small appreciation or even depreciation. In the later case, staying in the same house is the best choice for the house owners. (4) When the property tax rate increase \((t = 0.15)\), the situation is similar to (3) but the utility levels fall. (5) When the expenditure proportion on housing increases \((t = 0.02\text{ and } \alpha = 0.6)\), the situation is similar to the case (3) but the utility levels fall. (6) For a given housing appreciation, the higher property tax rate, the less advantages for people changing their houses. Thus, staying the same house is the best choice for house owners when the property tax rate is really high.

This model can embody a moving cost \((z)\). The only difference in modeling is equation (3) should be change to

\[
L = \hat{u} + \lambda (w' - p'(1 + t - \beta)h - c - z).
\]  

(9)

Since \(z\) is assumed constant. Therefore, the \(v'\) curve will vertically shift down and thus the owners will be better off only when appreciation or depreciation is bigger than those without \(z\). Details are available upon request to the authors.
3 Conclusion

Frank (2006) provides a paradoxical example which demonstrates that housing appreciation and depreciation both benefit owners. A more general model which includes property taxes, mortgage loan, and moving costs is constructed in the current paper. It is stressed that changing a house needs to pay a property tax and the previous mortgage loan should be repaid. Thus, it is shown that the Frank’s conclusion is valid only for those cases that no property tax, no mortgage loan, and no moving costs. With a property tax, a small change of housing price can never raise owners’ utility. When people buy their houses by using a mortgage loan, it is more possible to be worse off when housing price falls. This is very different from the conclusion in LMM model.

The policy implications are as follows: Housing appreciation is not always better for the owners who intends to adjust their house sizes when property tax is considered, especially for a small appreciation. This is because that the property tax is one kind of transaction cost for house owners. If the gain coming from appreciation can not cover the increase of property tax, the owner is still worse off. For the depreciation case, changing to a new house is not a best choice for the owners, not only from the property tax consideration, but also from a mortgage loan should be paid off before buying a new house whereas the old house is no longer so valuable. This result is close to the reality in the financial crisis during 2008. When housing market has a slump, people who buy their houses by using a mortgage loan feel worse off, instead of better off as predicted in Frank’s (2006) model. This is because that the owners can not pay off their mortgage loans and therefore can not change to a new house to maximize their utilities, and then they can only stay in the same house (enjoy a little utility increase from a lower property tax payment). Perhaps, when people have better to adjust their houses (by changing to a new house) but fail to do that, the housing market slump may thus be further enlarged.

References

